$\therefore\ dF=k\sqrt{2g(y-x)}dx$, where k=width of slit, and F=flow of water.

Whence $F=k\sqrt{2g}\int_0^y \sqrt{y-x}\ dx=\dfrac{2k\sqrt{2g}}{3}y^{\frac{3}{2}}$.

Call V the volume of water in the box at any instant.

Then $\dfrac{dV}{dt}=\dfrac{2k\sqrt{2g}}{3}y^{\frac{3}{2}}$. But $V=aby$, where a and b are the dimensions of base of box. $\therefore \dfrac{abdy}{dt}=\dfrac{2k\sqrt{2g}}{3}y^{\frac{3}{2}}$.

From which $t=\dfrac{3ab}{2k}\int_n^m y^{-\frac{3}{2}}dy=\dfrac{ab}{k\sqrt{2g}}\left[\dfrac{1}{\sqrt{n}}-\dfrac{1}{\sqrt{m}}\right]$, m and n being the depths of water at beginning and end of time of discharge.

If $n=0$, or the box is emptied, $t=\infty$.

If $m=\infty$, $t=\dfrac{ab}{k\sqrt{2gn}}$; or the time to empty a box of infinite depth to a finite depth is finite.

37. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

A thin board, of which the elements are given, is balanced at the center but inclined at an angle. A sphere of known dimensions is put directly above the point of suspension and liberated. Find the motion of the system. That is, find (a) the time until the sphere leaves the board, (b) the ultimate angular velocity of the board.

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Take the horizontal line through the point making the greatest angle with the plane in its initial position as the axis of x, and the axis of y vertically downward through the same point. Let R and T be the normal and tangential reactions of the plane and sphere at any time t from the commencement of motion, θ and ϕ the angles of rotation of the sphere and of the plane, m, k, a the mass, radius of gyration, and radius of the sphere, and r=the distance the sphere has moved on the plane, and x and y the coordinates of the center of the sphere.

Resolving horizontally and vertically, and taking moments about the center of the sphere,

$m\dfrac{d^2x}{dt^2}=R\sin\phi-T\cos\phi$.........(1), $m\dfrac{d^2g}{dt^2}=mg-R\cos\phi-T\sin\phi$ ....(2).

$mk^2\dfrac{d^2\theta}{dt^2}=Ta$..............(3). We also have $\theta=\dfrac{r}{a}+\phi$............(4),

$x=r\cos\phi+a\sin\phi$...............(5), and $y=r\sin\phi-a\cos\phi$...............(6).

Eliminating T from (1) and (2), $\sin\phi\frac{d^2x}{dt^2}-\cos\phi\frac{d^2y}{dt^2}=\frac{R}{m}-g\cos\phi$ .... (7).

Eliminating T and R from (1), (2), and (3),

$$\cos\phi\frac{d^2x}{dt^2}+\sin\phi\frac{d^2y}{dt^2}=g\sin\phi-\frac{k^2}{a}\frac{d^2\theta}{dt^2} \quad \ldots\ldots (8).$$

From (4), $\frac{d^2\theta}{dt^2}=\frac{1}{a}\frac{d^2r}{dt^2}+\frac{d^2\phi}{dt^2}$ .... (9).

From (5) and (6), $$\frac{d^2x}{dt^2}=\cos\phi\frac{d^2r}{dt^2}-2\sin\phi\frac{dr}{dt}\frac{d\phi}{dt}-r\cos\phi\frac{d\phi^2}{dt^2}$$

$$-r\sin\phi\frac{d^2\phi}{dt^2}-a\sin\phi\frac{d\phi^2}{dt^2}+a\cos\phi\frac{d^2\phi}{dt^2} \quad \ldots\ldots (10),$$

$$\frac{d^2y}{dt^2}=\sin\phi\frac{d^2r}{dt^2}+2\cos\phi\frac{dr}{dt}\frac{d\phi}{dt}-r\sin\phi\frac{d\phi^2}{dt^2}$$

$$+r\cos\phi\frac{d^2\phi}{dt^2}+a\cos\phi\frac{d\phi^2}{dt^2}+a\sin\phi\frac{d^2\phi}{dt^2} \quad \ldots\ldots (11).$$

Eliminating $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$, $\frac{d^2\phi}{dt^2}$ from (7) and (8),

$$2\frac{dr}{dt}\frac{d\phi}{dt}+r\frac{d^2\phi}{dt^2}+a\frac{d\phi^2}{dt^2}=g\cos\phi-\frac{R}{m} \quad \ldots\ldots (12),$$

$$\frac{a^2+k^2}{a^2}\frac{d^2r}{dt^2}-r\frac{d\phi^2}{dt^2}+\frac{a^2+k^2}{a^2}\frac{d^2\phi}{dt^2}=g\sin\phi \quad \ldots\ldots (13).$$

(12) and (13) seem to indicate that one more condition at least should be given.

PROBLEMS.

46. Proposed by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

"There was an old woman tossed up in a basket
Ninety times as high as the moon." *Mother Goose.*

Neglecting the resistance of the air, how long did it take the old lady to go up?

47. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

What is the focus of the convex surface of a plano-convex lens, index μ, which will converge parallel monochromatic rays to a given focus, the rays entering the lens on the plane side?